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Distinguishing Ordinal and Disordinal Interactions

Keith F. Widaman, Jonathan L. Helm,
Laura Castro-Schilo, and Michael Pluess
University of California, Davis

Michael C. Stallings
University of Colorado

Jay Belsky
University of California, Davis, King Abdulaziz University, and Birkbeck University of London

Re-parameterized regression models may enable tests of crucial theoretical predictions involving interactive effects of predictors that cannot be tested directly using standard approaches. First, we present a re-parameterized regression model for the Linear × Linear interaction of 2 quantitative predictors that yields point and interval estimates of 1 key parameter—the crossover point of predicted values—and leaves certain other parameters unchanged. We explain how resulting parameter estimates provide direct evidence for distinguishing ordinal from disordinal interactions. We generalize the re-parameterized model to Linear × Qualitative interactions, where the qualitative variable may have 2 or 3 categories, and then describe how to modify the re-parameterized model to test moderating effects. To illustrate our new approach, we fit alternate models to social skills data on 438 participants in the National Institute of Child Health and Human Development Study of Early Child Care. The re-parameterized regression model had point and interval estimates of the crossover point that fell near the mean on the continuous environment measure. The disordinal form of the interaction supported 1 theoretical model—differential-susceptibility—over a competing model that predicted an ordinal interaction.

Keywords: multiple regression, interactions, GXE interaction, differential-susceptibility, diathesis-stress

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Methods for testing interactive effects of predictors using multiple regression analysis are widely known and used. Several excellent texts (e.g., Aiken & West, 1991; Cohen, Cohen, West, & Aiken, 2003) discuss how to test Quantitative × Quantitative, Quantitative × Qualitative, or Qualitative × Qualitative interactions. If a significant interaction is detected, follow-up analyses are typically required to characterize the nature of the interaction, such as whether the interaction is ordinal or disordinal. A re-parameterized regression model that distinguishes clearly between ordinal and disordinal interactions and obviates the need for involved follow-up calculations to determine point and interval estimates of key parameters would be a useful adjunct to standard approaches. Here we propose such an approach and illustrate it using data for Gene × Environment (GXE) interactions. Although we selected GXE data for the demonstration, the approach advocated herein is general in nature and thus is applicable to a wide range of research domains in which statistical interactions are evaluated using regression analysis.

After discussing briefly why our new approach may be of use, we show how a linear regression model with a Linear × Linear interaction of two predictors can be re-parameterized to estimate parameters that characterize the ordinal or disordinal nature of the interaction and then adapt this approach to Qualitative × Quantitative interactions. We also apply our approach to a set of relevant data to demonstrate the unique outcomes obtained using our modeling approach.

Statistical Interactions in Substantive Research

Our efforts here were motivated by the fact that researchers often formulate interaction hypotheses imprecisely. If interaction hypotheses are phrased nonspecifically, misfit between theoretical formulations and trends in data may go unrecognized. If methods for testing specific interaction hypotheses were developed, re-

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Keith F. Widaman, Jonathan L. Helm, and Laura Castro-Schilo, Department of Psychology, University of California, Davis; Michael Pluess, Department of Human and Community Development, University of California, Davis; Michael C. Stallings, Department of Psychology, University of Colorado; Jay Belsky, Department of Human and Community Development, University of California, Davis, Department of Special Education, King Abdulaziz University, Jeddah, Saudi Arabia, and Department of Psychology, Birkbeck University of London, London, England.

Michael Pluess is now at the Institute of Psychiatry, King's College London, London, England.

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Correspondence concerning this article should be addressed to Keith F. Widaman, Department of Psychology, University of California, One Shields Avenue, Davis, CA 95616. E-mail: kfwidaman@ucdavis.edu

searchers could be challenged to provide more detail regarding the expected form of interactions. Without clear predictions, no definitive evidence regarding confirmation or disconfirmation of theoretical predictions is generated, aside from statistical significance of the interaction effect. Indeed, researchers often present disordinal interaction plots that appear inconsistent with their theories, but theory-data mismatch is rarely, if ever, noted. Armed with clearer predictions, misfit between predictions and results might be more readily recognized, leading to the need to revise theories to accord better with data.

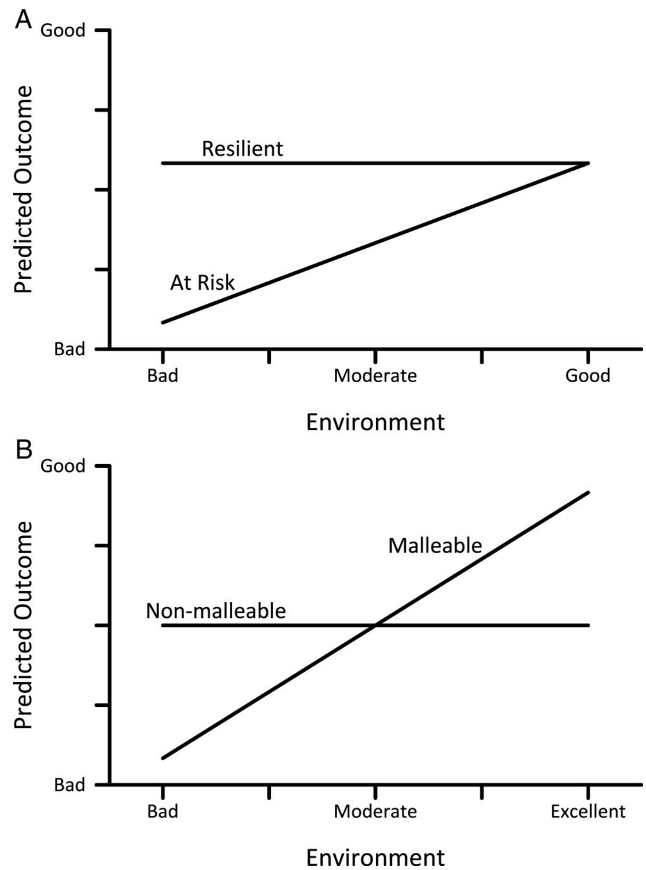

Figure 1. Predicted outcomes of Gene × Environment interaction under diathesis-stress (A) and differential-susceptibility (B).

One limitation of most research investigating interaction effects is lack of detail regarding the predicted *form* of the interaction. Researchers could specify whether an ordinal or disordinal interaction is predicted. For example, educational researchers might want to estimate the age at which one early intervention treatment becomes more effective than another, so policy makers can tailor interventions to children of appropriate ages. Or, Lynn (1999) offered a controversial maturational theory of intellectual development that holds that earlier maturation in females will lead to higher performance relative to males on intelligence tests at early ages. But by mid to late adolescence, males will begin to outperform females due to their later maturation and larger brain size. Research contexts such as these suggest that interactions should be disordinal, with a crossover point at some point on age.

One domain in which specific forms of interaction differentiate theoretical positions is the study of GXE interactions. Many GXE studies (e.g., Caspi et al., 2002, 2003) are based on a *diathesis-stress* model of environmental action (Belsky et al., 2009). Under diathesis-stress (Zuckerman, 1999), individuals with a "risk or vulnerability" gene are affected negatively by poor environments, whereas individuals with a different version of the same gene are relatively unaffected by environments. In the best environments, persons with differing polymorphisms may exhibit similar levels of behavior, but behavior of the groups diverges with worsening environmental conditions. Diathesis-stress therefore leads to prediction of a GXE interaction with the *ordinal* form shown in Figure 1A.

Recently, two research teams advanced a different theoretical model, *differential-susceptibility* (Belsky, 1997, 2005; Boyce & Ellis, 2005; Ellis, Boyce, Belsky, Bakermans-Kranenburg & Van IJzendoorn, 2011). Differential-susceptibility also leads to prediction of a GXE interaction but one *disordinal* in form. Under differential-susceptibility, persons carrying a so-called risk allele may simply be more *malleable.* From this perspective (and in accord with diathesis-stress), persons with a putative high-risk allele should exhibit poorer outcomes in poor environments and similar outcomes to persons with a low-risk allele in average environments. However, the model suggests that, in very good environments, persons with a putative high-risk allele will show outcomes that are superior to persons with the low-risk allele. This theoretical conceptualization leads to prediction of the *disordinal*, or crossover, interaction in Figure 1B. Thus, diathesis-stress and differential-susceptibility theories make identical predictions about the differing slopes for the two gene allele groups; what distinguishes predictions under the two models is the location of the crossover point (cf. Figure 1).

Methods for testing interactions reflecting differential-susceptibility have been proposed (e.g., Belsky, Bakermans-Kranenburg, & van IJzendoorn, 2007; Belsky & Pluess, 2009; Ellis et al., 2011) and applied to GXE data (e.g., Bakermans-Kranenburg, Van IJzendoorn, Pijlman, Mesman, & Juffer, 2008). Our goal is to develop a more direct test of competing predictions regarding the ordinal versus disordinal nature of an interaction that is widely applicable across research domains, including GXE studies.

Regression Equations With a Linear × Linear Interaction

Standard Parameterizations

A linear regression model with a Linear × Linear interaction can be written as

$$Y_i = B_0 + B_1 X_{1i} + B_2 X_{2i} + B_3 (X_{1i} \cdot X_{2i}) + E_i, \tag{1}$$

where Y_i is the score of person i ($i = 1, \ldots, N$) on the dependent variable; B_0 is the intercept; the B_j ($j = 1, 2, 3$) are regression weights for the three predictors; X_{1i} and X_{2i} are scores of person i on predictors X_1 and X_2, respectively; and E_i is a stochastic error score. The third predictor in Equation 1 is the product of X_{1i} and X_{2i} and carries the interactive effect of X_{1i} and X_{2i} if the two lower order effects (i.e., X_{1i} and X_{2i}) are included in the equation (cf. Cohen, 1978).[1]

Equation 1 can be fit using raw scores on X_1 and X_2, but regression coefficients and standard errors for X_1 and X_2 can be rather volatile if the product term is in the equation. To reduce these problems, many experts (e.g., Cohen et al., 2003) recommend centering X_1 and X_2 at their respective means, leading to:

$$Y = B_0^* + B_1^* X_1^* + B_2^* X_2^* + B_3^*\left(X_1^* \cdot X_2^*\right) + E, \tag{2}$$

where X_1^* and X_2^* are sample-mean-centered versions of X_1 and X_2, respectively; asterisks on B_0^* through B_3^* indicate weights for mean-centered predictors; and other symbols were defined above. Sample-mean-centering often reduces correlations among predictors and leads to many interpretive advantages (see, e.g., Aiken & West, 1991; Cleary & Kessler, 1982).

A Linear × Linear interaction effect of X_1 and X_2 on a quantitative outcome variable Y can assume various forms. But one feature of all Linear × Linear interactions is that predicted values from the fitted equation for different values of X_2 converge to a single crossover point at some point on X_1, if predicted values are projected onto the (Y, X_1) plane. Of course, predicted values for X_1 converge to a single crossover point at some value of X_2 if predicted values are projected onto the (Y, X_2) plane.

Placement of the crossover point has led researchers to distinguish between ordinal and disordinal interactions. In brief, *an ordinal interaction has the crossover of predicted values at the boundary* (e.g., Figure 1A) *or outside the range of observed values on* X_1 in the study (e.g., Figure 2A), whereas *a disordinal interaction contains a crossover of predicted values within the observed range of values on* X_1, as in Figures 1B and 2B. Therefore, the location of the crossover point is central to differentiating the two forms of linear interaction.

Consistent with Aiken and West (1991), we derived a point estimator for the crossover point as follows: Select two values for X_2 (e.g., 0 and 1), insert one value for X_2 into the right side of Equation 1, insert the other value for X_2 into the right side of Equation 1, set the two equations to equality, and solve for X_1:

$$B_0 + B_1 X_1 + B_2(0) + B_3(X_1 \cdot 0) = B_0 + B_1 X_1 + B_2(1) + B_3(X_1 \cdot 1), \tag{3}$$

which, after a little algebra, yields

$$X_1 = -\frac{B_2}{B_3} = C, \tag{4}$$

where C is a symbol for the crossover point, and other symbols were defined above.

An analog of Equation 4 can be obtained using mean-centered predictors. This solution is

$$X_1^* = -\frac{B_2^*}{B_3^*} = C^*, \tag{5}$$

which yields the crossover point C^* in a mean-centered metric. To calculate the crossover point in the raw metric of X_1, one must add $\overline{X}_1$ to each side of Equation 5, leading to

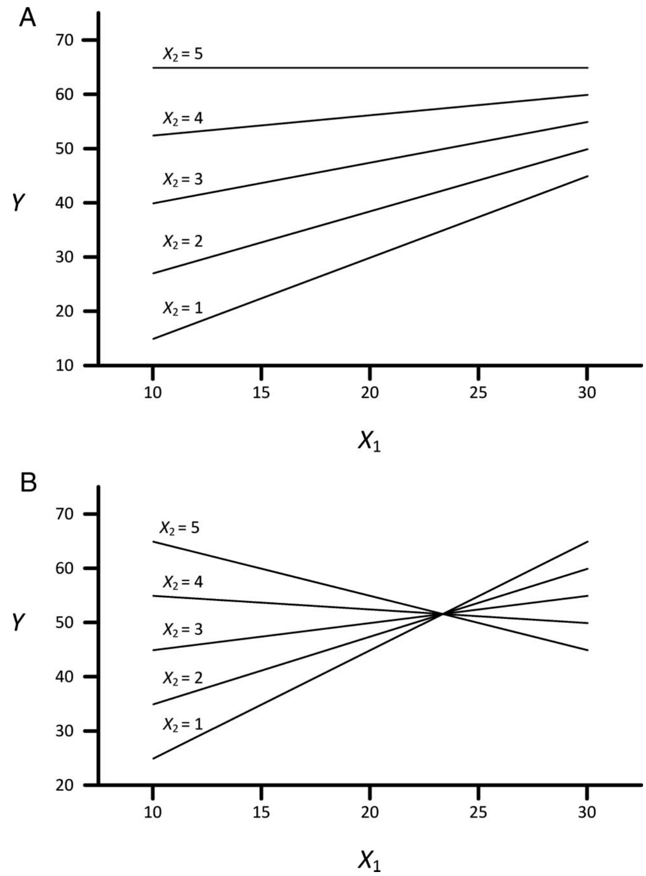

Figure 2. Plots of Linear × Linear interaction of two quantitative predictors X_1 and X_2: ordinal interaction (A) and disordinal interaction (B).

$$X_1 = -\frac{B_2^*}{B_3^*} + \overline{X}_1 = C, \tag{6}$$

where symbols in Equations 3–6 were defined previously (see Aiken & West, 1991, for details).

Re-Parameterized Equation

Derivation of re-parameterized model. Centering a predictor at its sample mean is a choice, with many advantages (Aiken & West, 1991; Cleary & Kessler, 1982), but not the only choice. We decided to center X_1 at C, the crossover point on X_1. This involved substituting $(X_1 - C)$ in place of X_1 in Equation 1. To determine the expected value of Y (or $\hat{Y}$) when X_1 is at the crossover point, we solved the following equation:

$$\mathrm{E}\left(Y_{X_1 = C}\right) = B_0 + B_1(C) + B_2(\theta) + B_3(C \cdot \theta), \tag{7}$$

where E() is the expected value operator, θ is any random value of X_2, and other symbols were defined above. Substituting Equation 4 into Equation 7 yields

[1] We used the i subscript for persons in Equation 1 for precision. In the remainder of the article, we typically drop the i subscript to simplify our notation and presentation but retain the subscript if context demands.

$$E(Y_{X_1=C}) = B_0 + B_1\left(-\frac{B_2}{B_3}\right) + B_2(\theta) + B_3\left(-\frac{B_2}{B_3}\cdot\theta\right), \quad (8)$$

which simplifies to

$$E(Y_{X_1=C}) = B_0 - \frac{B_1 B_2}{B_3} = A_0, \quad (9)$$

where A_0 represents the expected value of Y for $X_1 = C$, and other symbols were defined above.

Predicted values for varying values of X_2 are identical when $X_1 = C$, because predicted values fall at a single point for any value of X_2. We altered Equation 1 by replacing X_1 with ($X_1 - C$) and placing the new intercept (Equation 9) in the equation. In this model, B_2 becomes inestimable, because X_2 has no relation to Y at the crossover point on X_1. The re-parameterized equation thus becomes

$$Y_i = A_0 + B_1(X_{1i} - C) + B_3((X_{1i} - C)\cdot X_{2i}) + E_i, \quad (10)$$

where all symbols were defined previously. Equation 10 is a four-parameter equation, because C is now a parameter to be estimated, with the same number of free parameters as Equations 1 and 2. Symbols for B_1 and B_3 remain the same as in Equation 1, because these coefficients are unchanged by re-centering X_1 at C. Equation 10 is a re-parameterization of Equation 1 (as shown in supplemental material[2]) and thus leads to identical predicted values when plotting interactions. We also note that, because of its form, Equation 10 must be estimated using a nonlinear regression program, rather than a standard linear regression program.[2]

As shown above, a point estimate of the crossover point $\hat{C}$ is simple to compute using Equations 1 and 4 or Equations 2 and 6, but an interval estimate is more difficult to compute. Using Equation 10, the *SE* of $\hat{C}$ can be used to calculate an interval estimate (e.g., a 95% confidence interval [CI]); estimation of *SE*s of parameters in Equations 1, 2, and 10 is discussed in supplemental material.[3]

Regression Equations With Linear × Qualitative Interaction

The foregoing results hold for a Linear × Linear interaction of two quantitative predictors, but must be modified if one of the predictors is qualitative in nature. Here, we consider parameterizations with two-group and three-group qualitative variables.

Standard Parameterizations

Dichotomous grouping. If only two groups are used (e.g., low-risk vs. high-risk), the regression model is similar to Equation 1. Let X_1 represent the quantitative predictor, and D_2 a dummy variable (0 = group 1, and 1 = group 2). The standard regression model is

$$Y_i = B_0 + B_1 X_{1i} + B_2 D_{2i} + B_3(X_{1i}\cdot D_{2i}) + E_i, \quad (11)$$

where D_{2i} is the score of person i on dummy variable D_2, the subscript 2 on D_{2i} is a reminder that group 2 has the unit value on the dummy variable, and other symbols were defined above.

A mean-centered version of Equation11 can also be formulated as

$$Y = B_0^* + B_1^* X_1^* + B_2^* D_2 + B_3^*(X_1^*\cdot D_2) + E, \quad (12)$$

where asterisks on regression weights indicate they are for mean-centered predictors, and other symbols were defined above. Only the quantitative predictor X_1 was mean-centered; centering the dummy variable D_2 would lead to a less interpretable set of regression weights.

Ternary grouping. The standard parameterization of a regression model with a Linear × Qualitative interaction involving three groups on the latter variable is

$$Y = B_0 + B_1 X_1 + B_2 D_2 + B_3 D_3 + B_4(X_1\cdot D_2) + B_5(X_1\cdot D_3) + E, \quad (13)$$

where D_2 and D_3 are dummy variables with unit values for persons in Groups 2 and 3, respectively, and other symbols were defined above. In Equation 13, Group 1 is the reference group, and D_2 and D_3 allow one to determine whether Groups 2 and 3 differ from Group 1 in mean level (or intercept) or in moderation with X_1. A mean-centered version of Equation 13 is

$$Y = B_0^* + B_1^* X_1^* + B_2^* D_2 + B_3^* D_3 + B_4^*(X_1^*\cdot D_2) + B_5^*(X_1^*\cdot D_3) + E, \quad (14)$$

where all symbols were defined above. Again, only the quantitative predictor was mean-centered, to retain interpretive advantages of regression coefficients for the dummy variables.

Model comparisons to test lower level and interactive effects using Equations 13 or 14 are well known (cf. Cohen et al., 2003, pp. 308–316) so are not detailed here. But Equations 11–14 are only as informative about the ordinal or disordinal nature of the interaction as was true of Equations 1 and 2. Modified versions of Equations 4 and 6 could be developed to estimate crossover points for different groups, but *SE*s or CIs would still be unavailable for these estimates.

Re-Parameterized Equation

Dichotomous grouping. A more directly informative understanding of a Linear × Qualitative interaction is obtained using the re-parameterized equation:

$$Y = A_0 + B_1(X_1 - C) + B_3((X_1 - C)\cdot D_2) + E, \quad (15)$$

where all symbols were defined above. The following equation is an equivalent formulation:

$$Y: \begin{cases} \text{group} = 1 & Y = A_0 + B_1(X_1 - C) + E \\ \text{group} = 2 & Y = A_0 + B_2(X_1 - C) + E \end{cases}, \quad (16)$$

where B_1 and B_2 are slopes on X_1 for Groups 1 and 2, respectively,

[2] Technical details demonstrating the equivalence of the standard and re-parameterized regression models are contained in the online supplemental materials. The supplemental materials also include syntax for carrying out these analyses in nonlinear regression programs in SAS, R, and SPSS.

[3] Given constraints of space, examples of fitting Equations 1, 2, and 10 to data with Linear × Linear interaction of two quantitative predictors—both ordinal and disordinal interactions shown in Figures 2A and 2B, respectively—could not be included in the present article. The online supplemental materials demonstrate the fitting of models and interpretation of results for data with ordinal and disordinal interactions. The supplemental material also provides details regarding estimation of *SE*s of parameter estimates in the alternative models considered in this article.

and other symbols were defined above. Equations 15 and 16 lead to exactly the same R^2 as Equations 11 and 12. Thus, Equations 11, 12, 15, and 16 are equivalent regression models, with the same number of free parameters and the same R^2. But Equations 15 and 16 have a unique advantage over Equations 11 or 12: the direct estimate for the crossover point C and its *SE*. The difference between Equations 15 and 16 is the way in which the slope on X_1 for Group 2 is represented. In Equation 15, B_3 is the difference between slopes on X_1 for Groups 1 and 2, so the slope for Group 2 must be calculated as $B_1 + B_3$; in Equation 16, B_2 is a direct estimate of the slope on X_1 for Group 2.

Ternary grouping. If the qualitative variable represents the presence of three groups, one modified version of Equation 13 can be written as

$$Y: \begin{cases} \text{group} = 1 & Y = A_0 + B_1(X_1 - C) + E \\ \text{group} = 2 & Y = A_0 + B_2(X_1 - C) + E \\ \text{group} = 3 & Y = A_0 + B_3(X_1 - C) + E, \end{cases} \tag{17}$$

where B_1 through B_3 are regression slopes on X_1 for Groups 1 through 3, respectively, and other terms were defined above. Equation 17 contains a single crossover or convergence point C, so is a restricted re-parameterization of Equation 13. That is, Equation 17 has 5 free parameters, whereas Equation 13 has 6 free parameters. Several alterations could be made to Equation 17 to introduce an additional parameter; for example, one could fit the following model:

$$Y: \begin{cases} \text{group} = 1 & Y = A_0 + B_1(X_1 - C_{12}) + E \\ \text{group} = 2 & Y = A_0 + B_2(X_1 - C_{12}) + E \\ \text{group} = 3 & Y = (A_0 + B_1(C_{13} - C_{12})) + B_3(X_1 - C_{13}) + E, \end{cases} \tag{18}$$

where C_{12} (labeled simply C in Equation 17) and C_{13} are the points at which regression lines for Groups 2 and 3, respectively, cross the line for Group 1, and other symbols were defined above. With the additional parameter, Equation 18 has the same number of free parameters and R^2 as Equation 13. Thus, a nested-model test of the difference in R^2 for Equations 17 and 18 provides a 1 *df* test of the hypothesis that a single crossover point holds for Groups 1, 2, and 3.

Considerations Regarding Re-Parameterized Models

Assumptions Underlying Estimation of the Crossover Point

Using re-parameterized models to obtain interpretable point and interval estimates of C rests on standard assumptions for linear regression. Three important assumptions are (a) linearity of relations among variables, (b) equal measurement precision and equal intervals across the range of each variable, and (c) the observed range of X_1 corresponding closely to its population range. First, regarding linearity, the crossover point might be estimated in biased fashion if a linear model were fit to data with a quadratic relation between X_1 and Y. Screening for nonlinearities in relations among variables would allow a researcher to evaluate the seriousness of this issue for data under consideration.

Second, the assumption about measurement precision and intervals at all points on a dimension is also of key importance. If this assumption were incorrect, point and interval estimates of the crossover point could be biased. This concern is difficult to evaluate empirically, but must be borne in mind. Third, drawing firm conclusions about the ordinal or disordinal nature of the interaction presumes that the full population range on X_1 is observed in a study or at least considered. If range restriction on a predictor occurs, the range of values observed in a study is narrower than in the population. A crossover point that falls outside the range of X_1 values observed in a study but still falls within the population range of X_1 values may require special care when characterizing the interaction as ordinal or disordinal.

Finally, we note that none of the three assumptions is unique to our re-parameterized equations, but all apply with equal force to the standard parameterizations of regression models when they are used to obtain point estimates of C.

Strengths and Weaknesses of Re-Parameterized Equations

Some strengths and weaknesses of our re-parameterized models deserve mention. One strength, already noted, is the ready calculation of an interval estimate for the crossover point. The *SE* that accompanies the point estimate of $\hat{C}$ allows one to calculate an interval estimate of $\hat{C}$, enabling a more nuanced evaluation of the form of the interaction.

This strength leads, however, to a potential complication when interpreting results. Four outcomes of point and interval estimates might be considered: (a) disordinal interaction (i.e., $\hat{C}$ falling within the range of X_1), with the entire CI for $\hat{C}$ falling within the observed (or potential) range of X_1; (b) disordinal interaction but with the CI for $\hat{C}$ falling partly outside the range of X_1; (c) ordinal interaction (i.e., $\hat{C}$ falling outside the range of X_1) but with the CI for $\hat{C}$ falling partly within the range of X_1; and (d) ordinal interaction, with the CI for $\hat{C}$ falling completely outside the range of X_1. Scenarios (a) and (d) allow clear interpretation: Under (a), both point and interval estimates of $\hat{C}$ are consistent with the interaction being characterized as disordinal; under (d), both point and interval estimates of $\hat{C}$ are consistent with the interaction being characterized as ordinal. Scenarios (b) and (c) are more problematic for interpretation. Under (b), one might conclude that the interaction is disordinal in the sample, but an ordinal interaction in the population cannot be rejected. In turn, (c) might be rendered as an ordinal interaction in the sample, but a disordinal interaction in the population cannot be rejected. Note that these complications arise only with consideration of the CI of $\hat{C}$. If a researcher used Equation 1 or 2 and calculated only the point estimate of $\hat{C}$, the result would be an overly simplified interpretation of the ordinal or disordinal nature of the interaction.

A second strength of the re-parameterized equation is the potential for modifying an equation to test additional, specific hypotheses regarding parameters describing the interaction. For example, consider a dichotomous variable S (i.e., a dummy variable for sex, coded 1 = male, 0 = female). Equation 10 could be modified in the following fashion:

$$Y = (A_0 + A_{0s}S) + (B_1 + B_{1s}S)(X_1 - (C + C_sS)) + (B_3 + B_{3s}S) \times ((X_1 - (C + C_sS)) \cdot X_2) + E, \tag{19}$$

where A_0 is the intercept for females, A_{0s} the intercept difference for males, B_1 is the slope of X_1 for females, B_{1s} the difference in X_1 slope for males, C is the crossover point for females, C_s the difference in crossover points for males, B_3 is the slope coefficient for the product term for females, and B_{3s} the difference in product term slope for males. One could test lower-order and interactive effects of sex by altering Equations 1 or 2 (see Cohen et al., 2003, for details). The resulting equation would have eight free parameters, just as Equation 19 does, and sex differences in the interaction would be embodied in coefficients. But point estimates of the crossover points for males and females still would not have *SE*s. In contrast, Equation 19 allows one to test specific hypotheses about sex differences in particular parameters, providing point and interval estimates of group differences on parameters that characterize the form of the interaction.

One possible weakness of re-parameterized models is the empirical identification of parameters for interactions with nil or small effect sizes. In the limit, if the interaction were completely absent, iterative fitting of model estimates would not converge and the estimate of the crossover point $\hat{C}$ would be empirically unidentified and inestimable; if the interaction coefficient were a very small positive or negative value, the crossover point $\hat{C}$ would be difficult to estimate and might tend to $\pm\infty$ with extremely large *SE*. Although some might view lack of convergence as a problem, it might be seen as a strength of the procedure, indicating that the interaction effect may be small or nonexistent. Or, if a test of an interaction were significant using a standard model, lack of convergence of a re-parameterized model may not be due to an extremely small interaction effect (e.g., one or more outliers may lead to nonconvergence), and the researcher should explore the data more fully to isolate the problem.

Empirical Example Using Data From the NICHD Child Care Study

To demonstrate the utility of the re-parameterized equation, we analyzed data from the National Institute of Child Health and Human Development (NICHD) Study of Early Child Care (NICHD-SECC). The NICHD-SECC was a 10-site study, with research sites across the United States (NICHD Early Child Care Research Network, 2005). A minimum of 100 participants was to be obtained at each site, and participating children and their mothers were enrolled in the study when children were 1 month of age.

Variables

We utilized data on child gene polymorphism, child sex, the quantitative variable of childcare quality, and the child outcome variable of social skills. The two-group gene categorization for this analysis was based on exon-3 VNTR in the dopamine D4 receptor gene (DRD4). Prior research (e.g., Bakermans-Kranenburg & van IJzendoorn, 2006; Belsky & Pluess, 2009) suggested that presence of a 7-repeat on DRD4 is a risk factor for many developmental outcomes. The dummy variable for DRD4 was coded as $D_2 = 0$ or 1 for absence or presence, respectively, of a 7-repeat. Of 438 participants with genotype data, 95 (22%) had the 7-repeat on DRD4, so constituted the high-risk/malleability group. The remaining 343 participants (78%) did not have the 7-repeat, so constituted the low-risk/malleability group. Child sex was coded as 0 = female, 1 = male; the sample was almost equally divided on sex (51.8% female).

The quantitative predictor was childcare quality, assessing more attentive, stimulating, and affectionate care and was derived from observational coding done at five times between child ages of 6 and 54 months. Sample statistics on childcare quality were $M =$ 2.83, $SD = 0.24$, $Mdn = 2.82$, and range 2.10–3.38. Children with a DRD4 7-repeat ($M = 2.87$, $SD = 0.24$) did not differ significantly on childcare quality from children without a 7-repeat ($M =$ 2.82, $SD = 0.24$), in either mean level, $t(439) = 1.74$, $p = .08$, or variability, $F(345, 94) = 1.03$, $p = .87$.

The outcome variable was teacher-reported social skills of children in Grade 1, assessed with the Social Skills Rating System (Gresham & Elliott, 1990). Standardized scores revealed sample mean and standard deviation ($M = 104.30$, $SD = 13.19$) that were near population values, indicating that children in the sample were fairly representative of the population. Greater detail on all measures is available in NICHD Early Child Care Research Network (2005).

GXE Results

As discussed earlier, a nonlinear relation between X_1 and Y can lead to bias in estimating the crossover point. As a preliminary analysis, we regressed Y on the linear, quadratic, and cubic trends of X_1 for each of the two groups. Using hierarchical testing, the quadratic and cubic trends had F-ratios of 0.13 and 0.02, respectively, for the DRD4 low-risk group, and F-ratios of 0.83 and 0.01, respectively, for the DRD4 high-risk group. These results suggest the absence of nonlinearities that might bias estimation of the crossover point under a linear specification.

Standard equations. First, we fit Equation 11 with raw-scored predictors (see left part of Table 1). The $X_1 \times$ Group interaction was significant, $\hat{B}_3 = 17.51$ ($SE = 6.23$), $p < .006$. The crossover point was estimated as $\hat{C} = -(-47.95)/17.51 = 2.74$, using Equation 4.

Then, we fit the mean-centered version of this equation, Equation 12, to the data (see middle section Table 1). The mean-centered equation gave the same estimate of the interaction effect and an identical estimate of crossover point, $\hat{C} = (-(1.56)/17.51) + 2.83 = 2.74$, using Equation 6, as with raw-scored predictors. Thus, both raw-scored and mean-centered equations provided evidence that the interaction was disordinal, with a point estimate of C close to the sample mean on X_1, although lack of a *SE* for the crossover point hinders full interpretation.

Re-parameterized equation. Next, we fit the re-parameterized Equation 16 to the data. Parameter estimates and their *SE*s and CIs are shown in the right side of Table 1. The point estimate of the crossover point, $\hat{C} = 2.74$ ($SE = 0.09$), 95% CI [2.55, 2.92], fell just below the sample mean on X_1 ($M = 2.83$). The lower limit of the CI for $\hat{C}$ fell 1.17 *SD* units below the sample mean of childcare quality and the upper limit fell 0.38 *SD* units above the sample mean, so the CI covers values in the middle of the range of X_1 in the sample. Thus, both point and interval estimates of the crossover point support a conclusion that the interaction was disordinal, providing stronger support for differential-susceptibility model than for diathesis-stress.

Table 1
Results for Standard and Re-Parameterized Regression Models for Social Skills: Data From the NICHD Study

Standard parameterization				Re-parameterized model		
Parameter	Raw score	Parameter	Mean-centered	Parameter	Crossover-centered	95% CI
B_0	94.92 (8.11)	B_0^*	104.57 (0.68)	A_0	104.30 (0.91)	[102.5, 106.0]
B_1	3.41 (2.87)	B_1^*	3.41 (2.87)	B_1	3.41 (2.87)	[−2.22, 9.05]
B_2	−47.95 (17.9)	B_2^*	1.56 (1.48)	C	2.74 (0.09)	[2.55, 2.92]
B_3	17.51 (6.23)	B_3^*	17.51 (6.23)	B_2	20.92 (5.53)	[10.05, 31.80]

Note. NICHD = National Institute of Child Health and Human Development; CI = confidence interval. Unless otherwise noted, tabled values are parameter estimates, with standard errors in parentheses. B_0 through B_3 are the intercept and three regression weights, respectively, using raw scored predictors; B_0^* through B_3^* are the intercept and three regression weights, respectively, using mean-centered predictors; and, for the re-parameterized equation (i.e., Equation 16), A_0 is the intercept; B_1 and B_2 are the slope coefficients for the effects of the crossover centered X_1 for Groups 1 and 2, respectively; and C is the crossover point.

A plot of predicted values of social skills for the two groups of children is shown in Figure 3. As predicted, childcare quality was nonsignificantly related to social skills for the low-malleability group, $\hat{B}_1 = 3.41$ ($SE = 2.87$). In contrast, childcare quality was relatively strongly and significantly related to social skills for the high-malleability group, $\hat{B}_2 = 20.92$ ($SE = 5.53$). Thus, at high levels of childcare quality, the high-malleability group had predicted levels of social skills that were higher than those for the low-malleability group; but, at low levels of childcare quality, the high-malleability group had lower predicted levels of social skills.

In supplemental analyses, we also tested whether child sex moderated results shown in Table 1. That is, we modified Equation 16 to include the effect of sex in a fashion analogous to that for Equation 19. Relative to females, males had a slightly lower estimated crossover point, $\hat{C}_s = -0.12$ ($SE = 0.24$), and a somewhat lower level of social skills at the crossover point, $\hat{A}_{0s} = -0.17$ ($SE = 1.84$). Also, males were slightly less affected than females by child care in both the low-malleability, $\hat{B}_{1s} = -2.37$ ($SE = 5.77$), and high-malleability groups, $\hat{B}_{1s} = -13.27$ ($SE = 11.10$). But, none of these effects was statistically significant, as t-values ranged between |0.41| and |1.20| (all $ps > .20$). Although accepting the null hypothesis can be a risky gambit, the present data provide no evidence that results differed significantly by child sex.

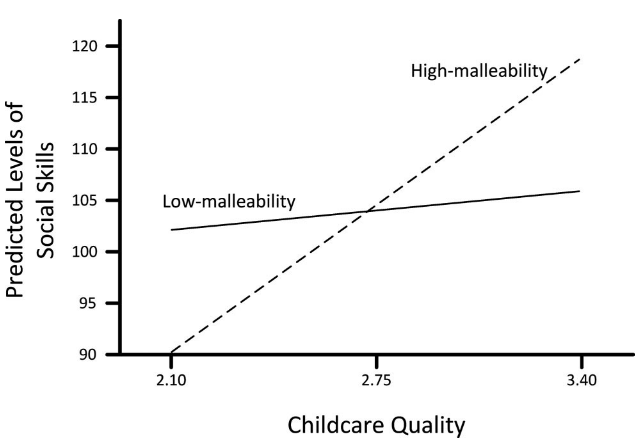

Figure 3. Predicted levels of social skills for the low-malleability and high-malleability groups as a function of childcare quality.

Discussion

Our primary aim was to re-parameterize the standard linear regression model to allow clearer distinctions between ordinal and disordinal interactions. Researchers often hypothesize interactive effects of predictors in fairly nonspecific terms. In our opinion, researchers should be challenged to make stronger predictions about the *form* of an interaction, such as whether the interaction is ordinal or disordinal. If such a prediction were warranted, then a re-parameterized regression model that estimates explicitly the crossover point of predicted values and its CI would enable stronger tests of the match between theoretical predictions and trends in data.

After presenting standard ways of parameterizing regression models with interaction effects, we derived a re-parameterized regression model for Linear × Linear interaction of two quantitative predictors. The most important benefit of a re-parameterized equation is the SE and associated CI of the estimated crossover point $\hat{C}$. Further, as stressed throughout, the CI of $\hat{C}$ allows a more informed evaluation of the ordinal versus disordinal form of the interaction.

Our procedures apply to any theoretically guided testing of interactions using regression analysis where the crossover point is at issue. In the context of GXE interactions, some have argued that negative emotionality, a quantitative temperament factor, is a diathesis, whereas others see it as a more general malleability marker (Belsky, 1997, 2005; Belsky & Pluess, 2009; Boyce & Ellis, 2005; Ellis et al., 2011). Thus, a researcher could use quantitative measures of both the environment (X_1 = childcare quality) and a genetically related factor (e.g., X_2 = negative emotionality) to test competing trends in Linear × Linear GXE interactions.

We extended the re-parameterization of the regression model to scenarios in which one of the interacting predictors is a categorical, or grouping, variable. If just two groups are present (e.g., low-risk vs. high-risk), only a single crossover point is possible. If a ternary, or three-class, categorization into groups is used, alternate models can test whether a single crossover point holds for all three groups or whether such a restriction should be rejected.

When we applied the standard and re-parameterized models to data on interactive effects of child-care quality and genotype (i.e., DRD4) on social skills, we found a significant GXE interaction. The disordinal form of the interaction was confirmed more strongly after fitting the re-parameterized model to the data by showing that both the point and interval estimates of the crossover point $\hat{C}$ were clearly within the range of values observed on the environmental variable. Further, the slope for the high-malleability group (i.e., DRD4–7R) was significant and the slope for the low-malleability group was nonsignificant, and both of these results proved consistent with tenets of the differential-susceptibility model.

Our proposed re-parameterized regression approach rests on critical assumptions and has some potential weaknesses that accompany its clear strengths. The assumptions are not unique to the new methods we proposed here but apply equally to use of standard approaches used to estimate the crossover point in interactions. Moreover, assumptions should always be evaluated to the extent possible. Threats to the validity of conclusions using any statistical procedures, our re-parameterized models included, should always be investigated, and conclusions should be qualified if assumptions are not fully met. In our opinion, the benefits of our re-parameterized equations outweigh any potential drawbacks to their use and supplement in informative ways traditional approaches to testing interactions using regression methods.

Our major goal was to develop a re-parameterized regression model that captures one essential aspect of an interaction more informatively than do standard analytic approaches. If the ordinal versus disordinal form of an interaction is crucial for distinguishing theoretical positions, our re-parameterized regression model yields more detailed information for evaluating the fit of data with theoretical predictions. With more useful tools for asking key questions, researchers can be challenged to provide more explicit hypotheses regarding predicted patterns in data. Confirming predicted patterns in data yields inductive support for the validity of a theory, but disconfirming predicted patterns points to the need to reconsider theory, measurements, or conditions to ferret out reasons for disconfirmation. Clearer predictions tested against data using more focused and definitive statistical models will provide clearer evidence regarding whether theoretical conjectures driving the research were confirmed or disconfirmed. We trust our re-parameterized equation will be yet one more tool for testing theoretical conjectures directly and strongly.

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